Exercise 7 – 27.11.2025 - Solution

Characterization of unsaturated geomaterials

PART A

You are asked to characterize the unsaturated behaviour of a soil. To do so, water retention measurements of this soil have been performed in the laboratory. The experimental results in terms of matric suction ($s = p_a - p_w$) and degree of saturation (Sr) are presented in the Excel file *"ex7Data.xls"*, tab *"PART A – data"*.

a) <u>Water retention curve</u>

From the experimental results, determine the best fitting parameters for the Van Genuchten water retention model:

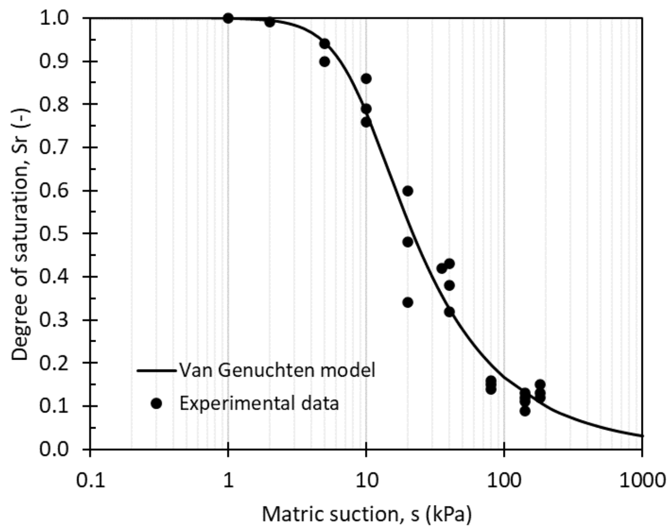
$$S_r = \left\{\frac{1}{1 + [\alpha(p_a - p_w)\,]^n}\right\}^m$$

α, *n* and *m* being the fitting parameters to determine. See the *annex 1* on how to determine those parameters using the Least Square Method in Excel. Plot the experimental and modelling results in the plane (*Sr – ln(s)*).

<u>ANSWER</u>

The best fitting parameters are the following: **α = 0.114 kPa^{-1}, n = 2.58 and m = 0.29.**

The following plot shows the experimental and modelling results in the plane (*Sr – ln(s)*).



More details on the problem solving are in *annex 1*.

b) <u>Permeability evolution</u>

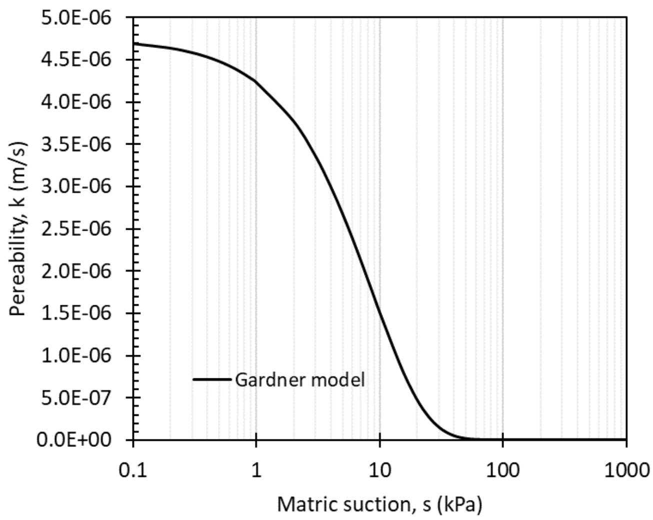
Using the previous results, plot the evolution of permeability (*k* [m/s]) according to Gardner's model in the plane (*k – ln(s)*).

$$k = k_{sat}e^{-\alpha(p_a - p_w)}$$

Assume the fitting parameter α to be the same as above, and the saturated permeability k_{sat} = $4.75 *10^{-6}$ *m/s*.

ANSWER

The following plot shows the evolution of the permeability according to the Gardner model.



c) Mechanical response

We want to analyse the change in mean effective stress (p') and the volumetric strain (ε_{vol}) during the drying process. The soil is subjected to zero mean total stress ($p = 0$), and (relative) air pressure is equal to zero as well ($p_a = 0$).We assume a linear elastic behaviour of the material, with a bulk modulus $K = 1'000$ *kPa*.

How the mean effective stress p' and the volumetric strain (ε_{vol}) evolve as function of matric suction s ? Plot the results in the planes ($p' – s$) and (ε_{vol} - s). Comment the results.

ANSWER

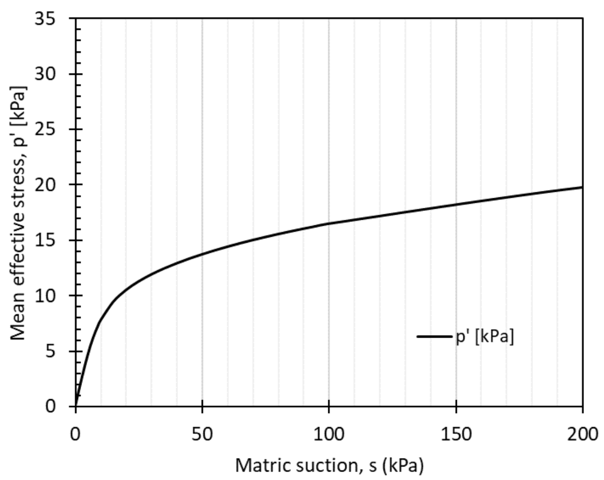
The effective stress for unsaturated soils can be expressed as follows:

$$p' = (p - p_a) + S_r s = p_{net} + S_r s$$

As the mean total stress p and the air pressure are both equal to 0, the mean net stress p_{net} is equal to 0 as well. Therefore, in the considered conditions, p' can be written as follow:

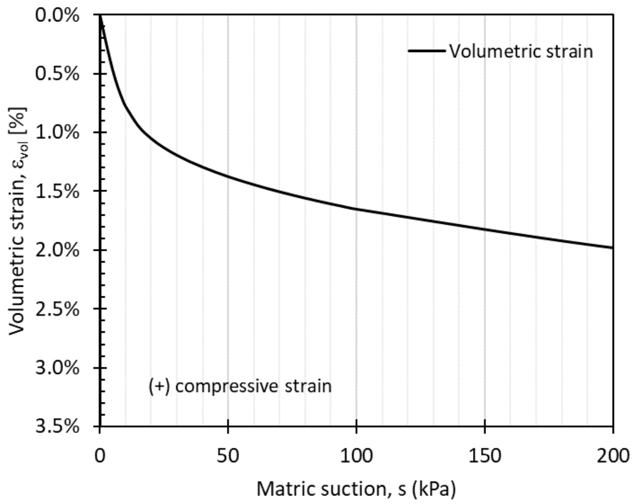
$$p' = S_r s$$

Using the results obtained in point (a) the mean effective stress p' as function of suction evolves as follows:



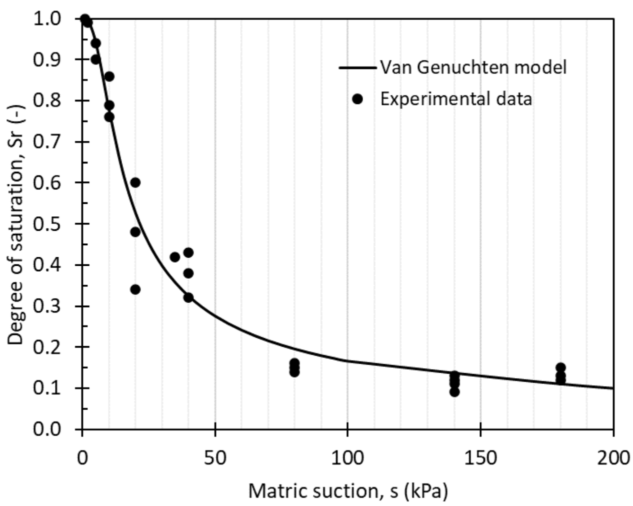
Assuming a linear elastic behaviour of the material, the relationship between the mean effective stress and the volumetric strain is as follows:

$$p' = K\varepsilon_{vol}$$



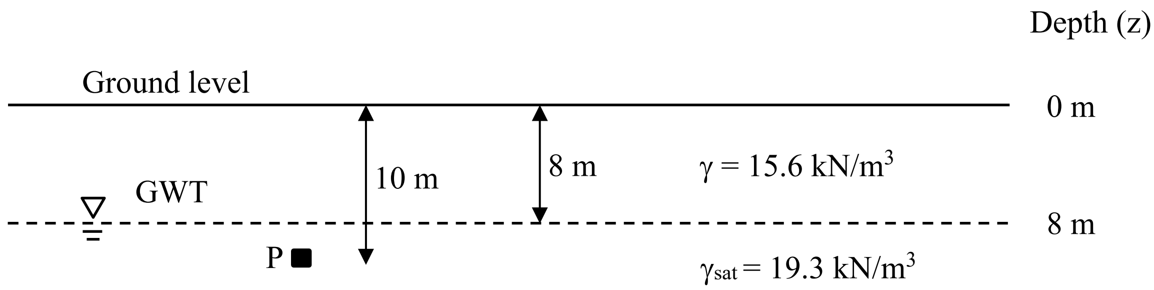
<u>Comments on the results :</u>

We can observe that most of the changes of the mean effective stress and volumetric strain occur at low matric suction (i.e. from 0 to 30 kPa of matric suction). The main reason is that in the considered conditions (p_{net} = 0), all changes in mean effective stress are caused by the suction stress (S_r*s). We can observe in the following plot that the degree of saturation decreases rapidly at low suction. As the degree of saturation becomes small, the change in effective stress is less pronounced at high suction values.



PART B

You are given a geotechnical project on a sandy soil layer with the following ground condition. During a site investigation, the ground water table (GWT) was found at 8 m depth. The dry and saturated bulk unit weights of the soil are reported as 15.6 kN/m^3 and 19.3 kN/m^3, respectively.



Due to a change in climate and subsequent lowering of the ground water table, the elevation of the GWT has been mobilized from 8 m to 11 m depth, and the soil at the point P became partially saturated. A geotechnical survey report suggests that the water content of a preserved sample cored adjacent to the point P at z = 10 m is w = 6.2%.

As a geotechnical engineer, you are asked to estimate the stress states of the soil at the point P, before and after the lowering of the GWT. Estimate the vertical stresses by answering the following questions.

Note: You will find the relevant data (suction – volumetric strain – water content) measured during a drying tets in the Excel file "ex7Data.xls", tab *"PART B – data".*

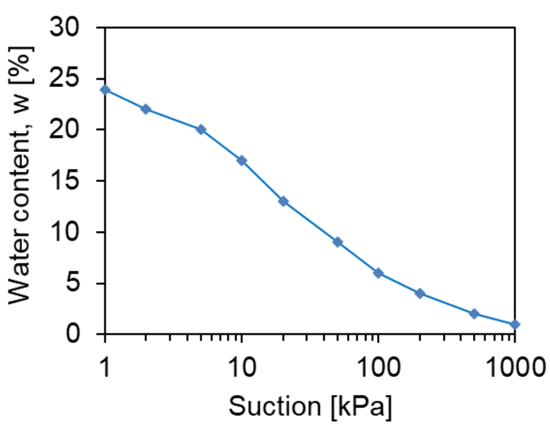
Question 1 – Saturated conditions (GWT is at z = 8 m)

1) Calculate the total and effective vertical stresses at the point P, before the lowering of the GWT.

Total stress: $\sigma = 15.6 \cdot 8 + 19.3 \cdot 2 = 163.4 \text{ kPa}$
Effective stress: $\sigma' = \sigma - p_w = 163.4 - 2 \cdot 10 = 143.4 \text{ kPa}$

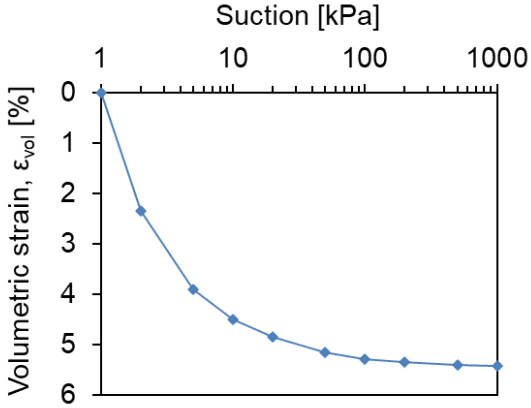
Question 2 – Partially saturated conditions (GWT is at z = 11 m)

2) Estimate the suction at the point P, after the lowering of the GWT, based on the water retention curve (drying path) that is expressing the suction (matric) and water content relationship.



From the water retention curve, the value of suction corresponding to w = 6.2% is around s = 100kPa.

3) Now consider the evolution of volumetric strain with respect to suction in the following figure.



Estimate the degree of saturation using the value of suction found in the previous step. Estimate also the air entry value of the material by plotting the entire water retention curve. For this, consider the following values to be representative of saturated conditions: suction s = 1 kPa, water content w_0 = 23.95%, initial void ratio e_0 = 0.60, initial porosity n_0 = 37.5% (assume grain density ρ_s = 2.5 g/cm^3).

In order to compute the degree of saturation, the void ratio is required. Due to the volumetric deformation experienced by the material during the drying process, the evolution of the void ratio (or porosity) with suction has to be taken into account (hydro-mechanical coupling).
At s = 100kPa, the volumetric deformation is ε_{vol} = 5.3%.

The corresponding void ratio is computed as follows (demonstrations of the formulas in the annexe):

$$n = \frac{n_0 - \varepsilon_{vol}}{1 - \varepsilon_{vol}} = \frac{0.375 - 0.053}{1 - 0.053} = 34.0\% \qquad e = \frac{n}{1-n} = 0.515$$

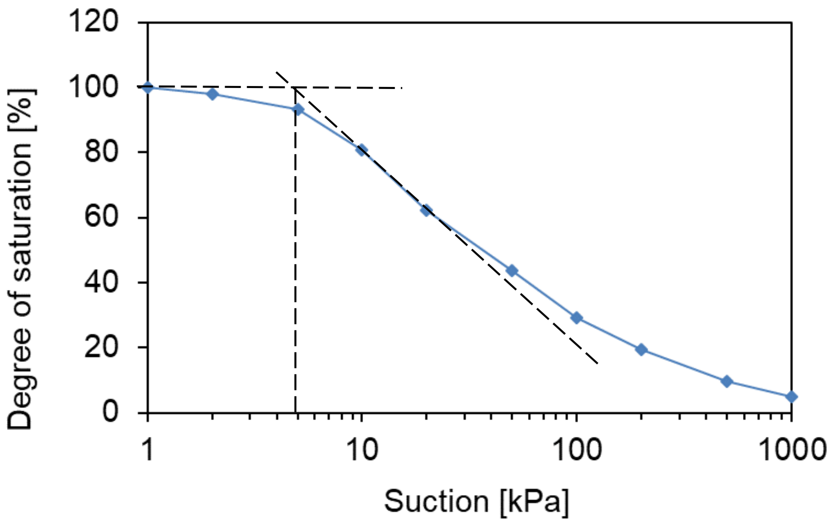
Alternatively, the void ratio can also be computed directly with the following formula:

$$e = e_0(1 - \varepsilon_{vol}) - \varepsilon_{vol} = 0.60(1 - 0.053) - 0.053 = 0.515$$

Finally, the degree of saturation corresponding to suction s = 100kPa is:

$$S_r = \frac{\gamma_s}{\gamma_w}\frac{w}{e} = \frac{25}{10}\cdot\frac{0.062}{0.515} = 30\%$$

The computation of the air entry value requires the definition of the drying curve in term of degree of saturation vs suction. By using the procedure just described above, it is possible to calculate the degree of saturation for different values of suction. The following graph can be obtained:



The air-entry value represents the value of suction at which the material starts to desaturate. Considering the water retention curve in the plane $s - Sr$, it is observed that the desaturation of the material begins at a suction around s = 3 to 6 kPa. This range of suction defines the air entry value.

4) Calculate the effective stress at the given depth of 10 m, using the expression $\sigma' = \sigma + s \cdot Sr$ (based on the generalized effective stress formulation, where σ is the total stress, s is the suction and Sr is the degree of saturation). Compare the obtained value of effective stress with the one obtained for the saturated condition (Question 1) and show the difference.

Given the water retention curve in terms of $s - S_r$, at z = 10 m we have that:

$$\sigma' = \sigma + s \cdot Sr = \gamma \cdot z + s \cdot Sr = \ 15.6 \cdot 10 + 100 \cdot 0.30 = 186 \text{ kPa}$$

Where we have assumed, approximately, a constant unit weight (equal to the dry one) for the soil under unsaturated conditions. The obtained effective stress results to be higher with respect to the one calculated for the saturated condition. This increase is due to the capillary forces that now act at the interface between air and water.

Question 3 – Discussions

5) As a geotechnical engineer, you are asked to provide an estimate of the shear strength of the soil at the point P. Did the shear strength of the soil has increased/decreased due to the lowering of the ground water table? Provide your judgment and show why you think so, based on the findings above.

The shear strength is supposed to have increased due to the lowering of the ground water table. This is because the effective stress of the soil has increased, even though the total stress is constant.

Annex 1

To obtain the best fitting parameters for the Van Genuchten model with respect to the experimental data, the Least Square Method is used. The objective consists of adjusting the parameters of a model function to best fit a data set. The fit of a model to a data point is measured by its residual, defined as the difference between the observed value of the dependent variable and the value predicted by the model:

$$r_i = y_i - f(x_i)$$

For each point *i, r* being the residual, *y* the dependant variable and *f(x)* the results of the model. In our case, $y_i = Sr_i$ is the degree of saturation obtained in the experiment at each matric suction ($x_i = s_i$). $f(x_i)$ is the computed degree of saturation at each step according to the Van Genuchten model.

The least-squares method finds the optimal parameter values by minimizing the sum of squared residuals, *S*:

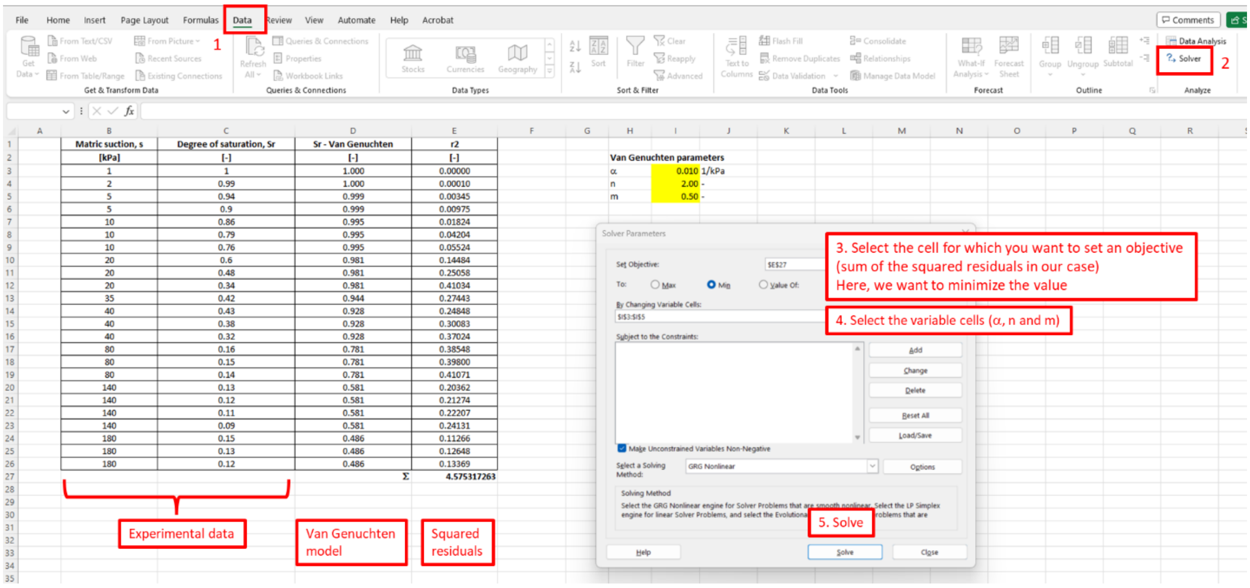
$$S = \sum_{i=1}^{n} r_i^2$$

To be able to fit the Van Genuchten parameters in Excel, the Solver Add-in has to be activated:

1) In Excel 2010 and later, go to **File > Options**
2) Click **Add-Ins**, and then in the **Manage** box, select **Excel Add-ins**.
3) Click **Go**.
4) In the **Add-Ins available** box, select the **Solver Add-in** check box, and then click **OK**
5) After you load the Solver Add-in, the **Solver** command is available in the **Analysis** group on the **Data** tab

You can then use the Excel solver to minimize the sum of squared residuals, S, and finding the best fitting parameters.

Do not start with zero values for α, n and m. You can start the iteration with α = 0.01 (kPa^{-1}), n = 2 and m = 0.5.



Annex 2

Final porosity as a function of the volumetric strain and of the initial porosity

$$n = \frac{V_v}{V} = \frac{n_0 V_0 - \varepsilon_v V_0}{V_0 - \varepsilon_v V_0} = \frac{n_0 - \varepsilon_v}{1 - \varepsilon_v}$$

In which: n is the final porosity, V_v is the final volume of the voids (air and water) n_0 is the initial porosity, V_0 is the initial total volume (air, water and solid) and ε_v is the volumetric strain.

By knowing the porosity, the computation of the void ratio is straightforward:

$$e = \frac{n}{1 - n}$$

In which: e is the void ratio

In fact:

$$e = \frac{V_v}{V_s} = \frac{V_v}{V}\frac{V}{V_s} = n\frac{V}{V - V_v} = \frac{n}{\frac{V - V_v}{V}} = \frac{n}{1 - n}$$

In which: V_v is the volume of the voids (air and water), V_s is the volume of the solid, V is the total volume, n is the porosity.

Final void ratio as a function of the volumetric strain and of the initial void ratio

$$e = \frac{V_{v0} - \varepsilon_v V_0}{V_s} = e_0 - \frac{\varepsilon_v V_0}{V_s} = e_0 - \frac{\varepsilon_v (V_{v0} + V_s)}{V_s} = e_0 - \varepsilon_v e_0 - \varepsilon_v$$

In which: e is the final void ratio, V_{v0} is the initial volume of the voids (air and water), V_0 is the initial total volume (air, water and solid), ε_v is the volumetric strain, V_s is the volume of the solid and e_0 is the initial void ratio.

Degree of saturation as a function of water content and void ratio

$$S_r = \frac{V_w}{V_v} = \frac{\frac{V_w}{V_s}}{\frac{V_v}{V_s}} = \frac{\left(\frac{m_w}{\rho_w}\right) \Big/ \left(\frac{m_s}{\rho_s}\right)}{\frac{V_v}{V_s}} = \frac{\frac{m_w}{m_s}\frac{\rho_s}{\rho_w}}{\frac{V_v}{V_s}} = \frac{\rho_s}{\rho_w}\frac{w}{e}$$

In which: V_w is the volume of the water, V_v is the volume of the voids (air and water), V_s is the volume of the solid, m_w is the mass of the water, m_s is the mass of the solid, ρ_s is the solid density, ρ_w is the water density, w is the water content and e is the void ratio.